

An introduction to Quasielastic Neutron Scattering (QENS)

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American Conference on Neutron Scattering, May 11-15, 2008; QENS Tutorial

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Outline

- A brief history of neutron scattering
- Scattering fundamentals
- What is QENS, and what does it look like?
- How is QENS measured?
- How is QENS modeled?
- Practicalities
- Examples of QENS
- Further reading
- The bottom line

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A brief history of neutron scattering

- Diffraction --- Shull and Wollan (late '40s)

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The original two axis spectrometer at the "Clinton Pile", Oak Ridge (critical 1943)

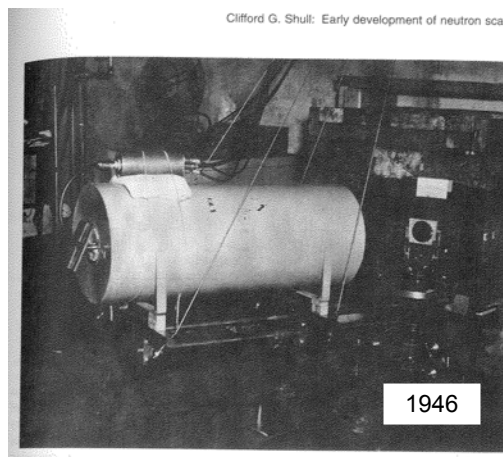


FIG. 3. Photograph of the first double-axis neutron spectrometer used to take powder-diffraction patterns at the Clinton Pile.

C.G. Shull, Rev. Mod. Phys. 67 (4) 753 (1995)
[Nobel lectures in physics 1994]

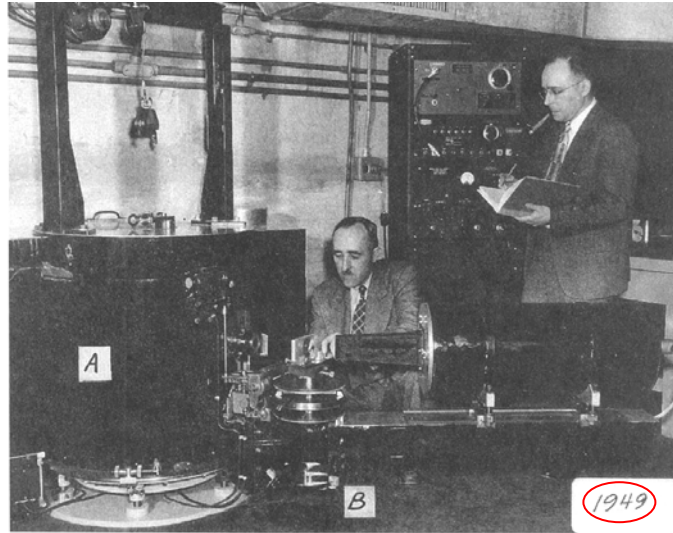
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Neutron diffractometer at the Graphite Reactor (Clinton Pile), ORNL

E.O. Wollan

C.G. Shull



C.G. Shull, Rev. Mod. Phys. 67 (4) 753 (1995)

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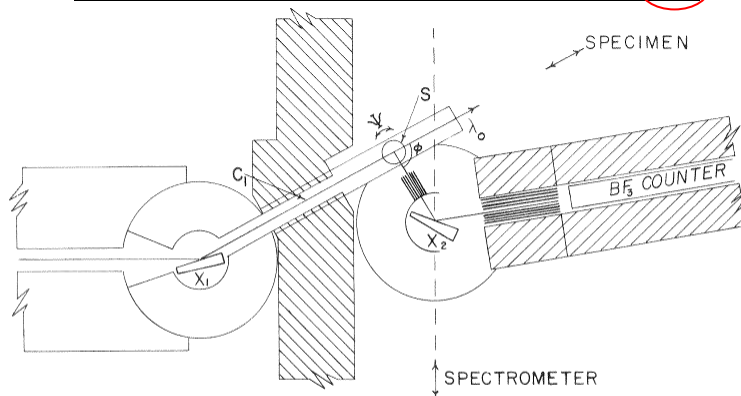
A brief history of neutron scattering

- Diffraction --- Shull and Wollan (late '40s)
- Inelastic scattering --- Brockhouse ('50s)
 - Absorbers
 - Triple axis, time-of-flight

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Triple axis spectrometer at NRX reactor, Chalk River, 1954



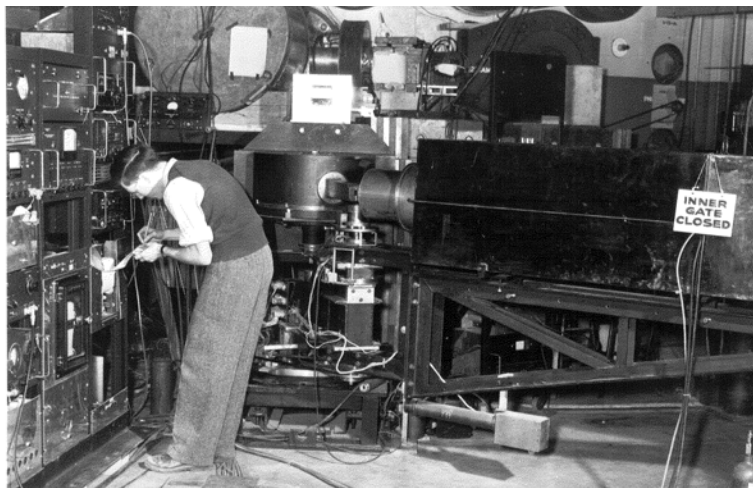
THE FIRST TRIPLE-AXIS CRYSTAL SPECTROMETER AT CHALK RIVER AT NRX REACTOR BUILT BY B.N. BROCKHOUSE IN 1954. NEUTRONS FROM THE REACTOR TO THE LEFT OF THE PICTURE ARE BRAGG SCATTERED BY A FANKUCHEN CUT MONOCHROMATOR. THIS ALUMINUM CRYSTAL IS MOUNTED AS THE ANALYSING CRYSTAL IN APPARATUS EXHIBITED.

B.N. Brockhouse, Rev. Mod. Phys. 67 (4) 735 (1995) [Nobel lectures in physics 1994]

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Neutron diffractometer at the NRX reactor, Chalk River (critical 1947)



Courtesy
W.J.L. Buyers

NEUTRON DIFFRACTION APPARATUS BUILT IN THE LATE FORTIES AT NRX REACTOR BY D.G. HURST WHO STARTED NEUTRON SCATTERING AT CHALK RIVER, N.Z. ALCOCK IS SHOWN READING A CHART. NOTE THE MODEST MONOCHROMATOR SHIELD AND HUGE DETECTOR SHIELD.

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A brief history of neutron scattering

- Diffraction --- Shull and Wollan (late '40s)
- Inelastic scattering --- Brockhouse ('50s)
 - Absorbers
 - Triple axis, time-of-flight
 - Phonons, spin waves, liquids, ...

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MINUTES OF THE 1955 ANNUAL MEETING HELD AT NEW YORK CITY, JANUARY 27-29, 1955
(Corresponding to *Bulletin of the American Physical Society*, Volume 30, No. 1)

M7. Slow Neutron Spectrometry—A New Tool for the Study of Energy Levels in Condensed Systems. B. N. BROCKHOUSE, *Chalk River Laboratories*.—It has long been realized that energy distributions of initially monoenergetic neutrons scattered by molecules or condensed systems would yield important information not otherwise easily accessible to experiment. With high reactor flux and suitable spectrometer design such measurements are now feasible. Monoenergetic neutrons, selected from the *NRX* reactor spectrum by a crystal, are scattered by the specimen and the energy distribution of the scattered neutrons is analyzed by another crystal. A survey of the field has been made and more detailed studies are in progress. The sharp division of incoherent scattering into elastic and inelastic components predicted by theory has been verified with vanadium. In liquid lead, light water, and heavy water a similar division into a "quasi-elastic" component, and an inelastic component which increases with increasing angle, can be made. For coherent scattering the "elastic" component is associated with the peaks in the liquid diffraction pattern. In liquid lead the inelastic component is similar to that of the solid at about the same temperature and angle of scattering. Diffuse magnetic scattering, studied with MnO and Mn_2O_3 , and with Cr_2O_3 above and below its Néel temperature, is largely inelastic, the mean energy changes being of the order of the Néel temperatures.

Invited Paper

M8. Study of Energy Levels in Solids, Liquids, and Magnetic Materials by Scattered Slow Neutrons. B. N. BROCKHOUSE, *Chalk River Laboratories*. (30 min.)

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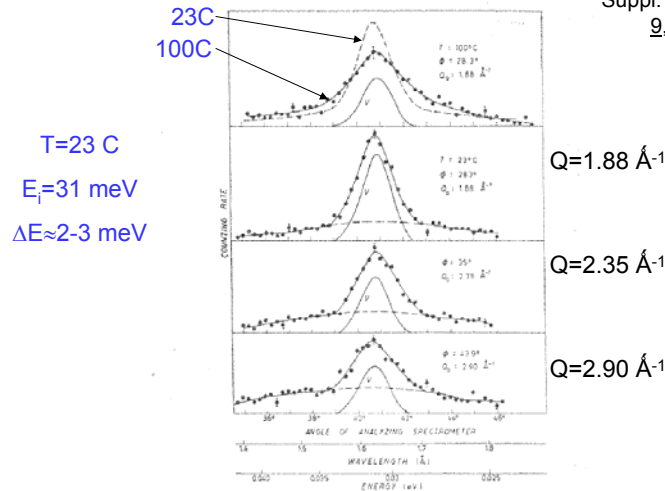
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Structural Dynamics of Water by Neutron Spectrometry (*).

B. N. BROCKHOUSE

General Physics Branch, Atomic Energy of Canada Limited - Chalk River, Ontario

Suppl. Nuovo Cimento
9, 45 (1958)



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A brief history of neutron scattering

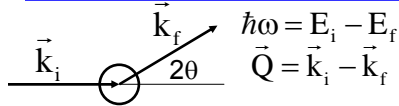
- Diffraction --- Shull and Wollan (late '40s)
- Inelastic scattering --- Brockhouse ('50s)
 - Absorbers
 - Triple axis, time-of-flight
 - Phonons, spin waves, liquids, ...
- High resolution instruments, high flux reactors ('60s, '70s),...
- **USERS**: chemists, biologists, etc etc

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Scattering fundamentals

Elastic and inelastic scattering



$$\hbar\omega = E_i - E_f$$

$$\vec{Q} = \vec{k}_i - \vec{k}_f$$

$$1 \text{ meV} \approx 0.24 \times 10^{12} \text{ c/s}$$

$$8.1 \text{ cm}^{-1}$$

$$11.6 \text{ K}$$

$$0.023$$

$$\text{kcal/mol}$$

$$0.10 \text{ kJ/mol}$$

Coherent and incoherent scattering

$$\sigma_{\text{coh}} = 4\pi \langle b \rangle^2 \quad \sigma_{\text{inc}} = 4\pi \left(\langle b^2 \rangle - \langle b \rangle^2 \right)$$

$$\text{Count rate } I = \Phi N \frac{d^2\sigma}{d\Omega dE_f} \Delta\Omega \Delta E_f$$

$$\frac{d^2\sigma}{d\Omega dE_f} = \frac{\sigma_{\text{coh}}}{4\pi\hbar} \frac{k_f}{k_i} S(Q, \omega) + \frac{\sigma_{\text{inc}}}{4\pi\hbar} \frac{k_f}{k_i} S_s(Q, \omega)$$

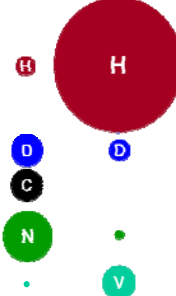
Correlation functions

$$I_s(\vec{Q}, t) = \int S_s(\vec{Q}, \omega) \exp(i\omega t) d\omega$$

$$G_s(\vec{r}, t) = \frac{1}{(2\pi)^3} \int I_s(\vec{Q}, t) \exp(-i\vec{Q} \cdot \vec{r}) d\vec{Q}$$

(similar expressions apply to I and S.)

Coh. Inc.



λ	E	v	τ
Å	meV	m/s	μs/mm
1	82	4000	0.25
2	20.5	2000	0.5
4	5.1	1000	1
8	1.3	500	2

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What is QENS, and what does it look like?

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What is QENS, and what does it look like?

- QENS is inelastic scattering that is almost elastic, centered at zero energy transfer
- There may or may not be associated elastic scattering
- QENS is associated with relaxation phenomena, such as translational diffusion, molecular reorientations, confined motion within a pore, hopping among sites, etc
- Accessible time scales range from fractions of ps to 100s of ns
- Length scales range from Å to 100s of Å
- Most QENS experiments are designed to study incoherent scattering (single particle motions)

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Examples of $S(Q, \omega)$ data

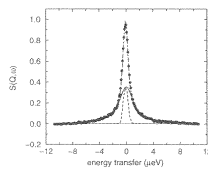


FIG. 2. Quasielastic scattering from water in NaA zeolite at 300 K, $Q = 1.32 \text{ \AA}^{-1}$. The curves are, from top to bottom, the total fitted intensity, the Lorentzian component broadened by resolution, and the shape of the resolution function. Uncertainties due to counting statistics are approximately the size of the symbols.

W.A.Kamitakahara and N. Wada,
PRE 77, 041503(2008)

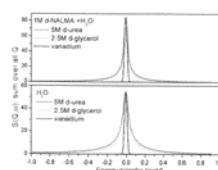


Fig. 2 The incoherent scattering function, summed over Q , for 1 M d-NALMA in H_2O (top) and in pure H_2O (bottom), together with 5 M deuterated urea and 2.5 M deuterated glycerol. The spectra are normalized to the maximum of 2.5 M d-glycerol data.

D. Russo, Chem. Phys. 345,200(2008)

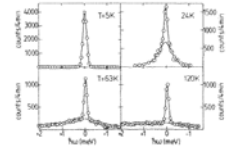


FIG. 5. Temperature dependence of the quasielastic neutron scattering spectrum from $Na_2SO_4/2H_2O$. The measurement at 9.8 Å is below the phase transition (19.9 K for the hydrogenated compound) and was used for the determination of the background level.

J. Eckert and W. Press, JCP 73(1)451(1980)

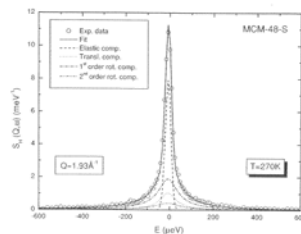


FIG. 2. A typical QENS spectrum from a MCM-41S hybrid sample at $T = 270 \text{ K}$, at $Q = 1.93 \text{ \AA}^{-1}$. The energy scale is larger than the resolution. The continuous line represents the overall fit, the dashed line is the elastic component, the dotted, dash-dot, and long-dashed lines represent contributions to the scattering from the first three terms of the fit expansion.

A. Faraone et al, JCP 119(7)3963(2003)

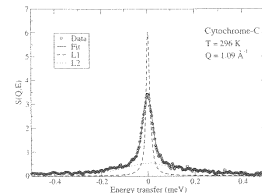


Fig. 7. Measured and calculated spectra for cytochrome-C at 296 K, with $Q = 1.09 \text{ \AA}^{-1}$. The experimental data, corrected for scattering from the buffer, are represented as points. The scatter in the points is an indication of their uncertainties. The fitted function (broadened by the instrumental resolution) is shown along with the two unbroadened Lorentzians, L1 and L2.

A.M. Pivovar (unpublished)

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More examples of $S(Q,\omega)$ data

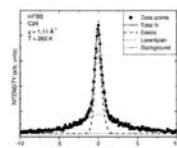


Fig. 1. Typical spectrum of a C20 micelle measured on the SFRS at a wave vector transfer $Q = 0.1 \text{ \AA}^{-1}$. The figure illustrates the decomposition of the intensity into an elastic component (dotted line) and a quasielastic Lorentzian component (solid line) with the same central scattering vector to give the rigid and soft modes of the micelle. The sample temperature is 300 K. Error bars give the statistical uncertainty (1- σ level).

A.D. Enevoldsen et al,
JCP 126, 104704 (2007)

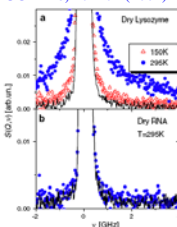


Fig. 2. Dynamic structure factor of dry lysozyme (a) and dry RNA (b). The solid line represents the correlation function. The spectra are summed up over all Q because no significant dependence of the spectral width has been observed. Error bars are of the order of the data scattering.

A.P. Sokolov et al,
Chem.Phys.345,212(2008)

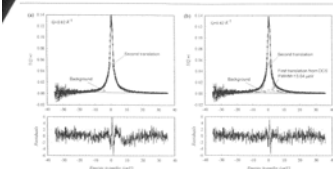


Fig. 3. Static and dynamic structure factors of a 1 M PAA solution at a wave vector of 0.1 \AA^{-1} on the SFRS at $T = 298 \text{ K}$ and $Q = 0.1 \text{ \AA}^{-1}$. (a) Static structure factor, (b) dynamic structure factor. The figure shows the experimental data as black circles, the static fit component as a continuous line, the dynamic component as a dashed line, and the total fit as a continuous line. The figure also shows the experimental data for the static structure factor as open circles.

C. Malardier-Jugroot and T. Head-Gordon,
PCCP 9, 1962 (2007)

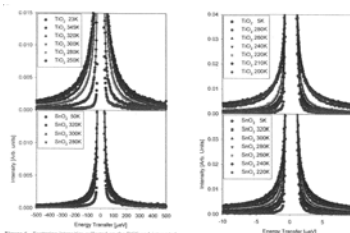


Figure 4. Scattering intensity collected on the SFRS and integrated over the range of the scattering vector transfer $0.40 \text{ \AA}^{-1} < Q < 0.50 \text{ \AA}^{-1}$ for long up (red) and down (blue) scattering.

E. Mamontov et al, J.Phys.Chem. C 111, 4328(2007)

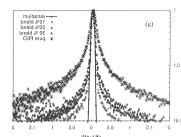


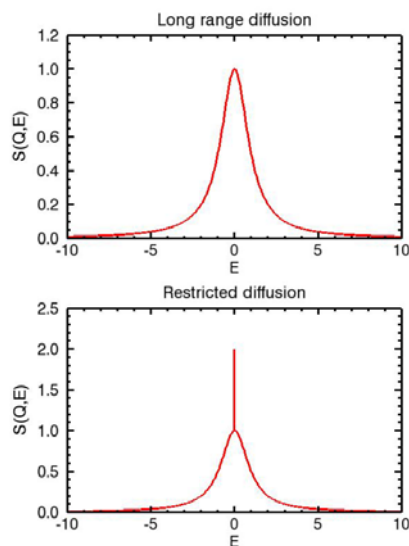
Figure 5. Scattering intensity collected on the SFRS and integrated over the range of the scattering vector transfer $0.42 \text{ \AA}^{-1} < Q < 0.50 \text{ \AA}^{-1}$ for long up (red) and down (blue) scattering.

V. Garcia Sakai, J.K. Maranas, et al, Macromolecules, in press.

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Typical QENS $S(Q,\omega)$



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Examples of $I(Q,t)$ data

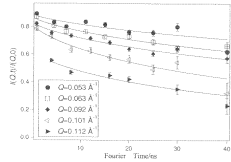


Figure 2. Decay of the normalized intermediate scattering function obtained by NSE as a function of Fourier time at different Q values for DOPC vesicles in D_2O at 60 °C.

M. Nagao et al, Proc. QENS 2006, p. 101
(Mater. Res. Soc. 2007)

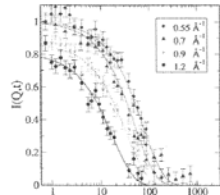


Figure 5. Incoherent intermediate scattering function $I(Q,t)$ obtained by NSE for hectorite clay in the monolayer (top) and bilayer (bottom) states at ambient temperature. Measured Q values are indicated in the legend. Lines correspond to best fits of the data with monoexponentials.

N. Malikova et al, J. Phys. Chem. C
111, 17603 (2007)

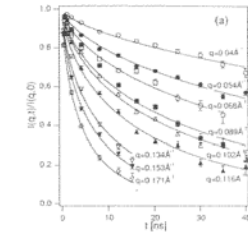
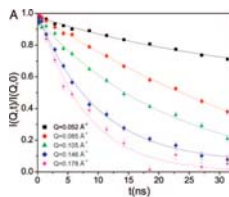


Figure 2: (a) Obtained $I(q,r)/I(q,0)$ at $T = 301.15$ K according to eq. (1). (b) Semi-logarithmic plot of $I(q,r)/I(q,0)$ versus r at $T = 301.15$ K. The solid lines show the existence of the correlation length ξ .



Z. Bu et al, PNAS 102, 17646 (2005)

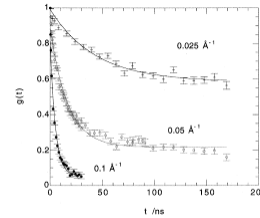


Figure 2. Neutron spin echo decays from a poly(fluorosilicone) gel swollen in acetone at $\varphi = 0.16$ for three different wave vectors q .

A.-M. Hecht et al, *Macromolecules*
35, 8552 (2002)

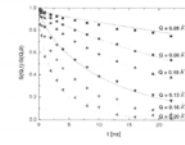


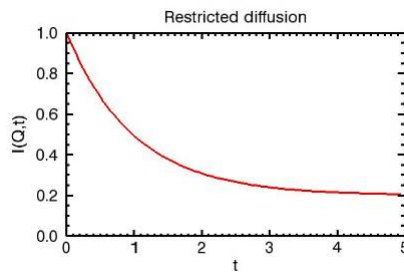
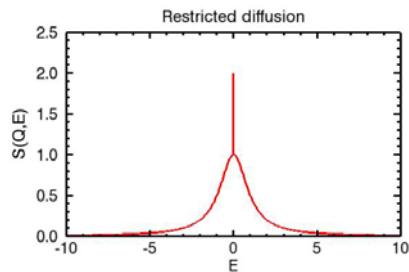
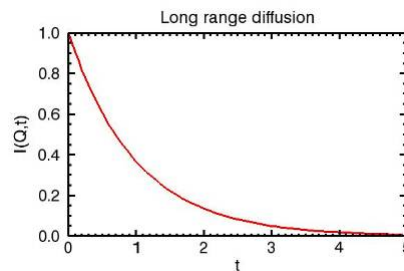
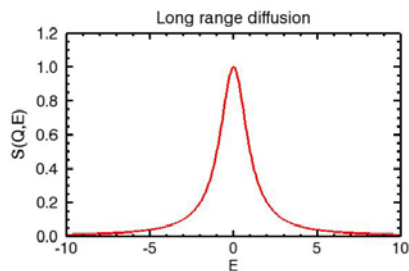
FIG. 2. NSE data of a PEO sample with $M_w = 2.4$ kg/mol at $T = 413$ K. Symbols show the data measured for various Q values. Solid lines represent a fit with the Rouse model simultaneously to

K. Niedzwiedz et al, submitted to
Macromolecules (2008)

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Typical $S(Q, \omega)$ and $I(Q, t)$



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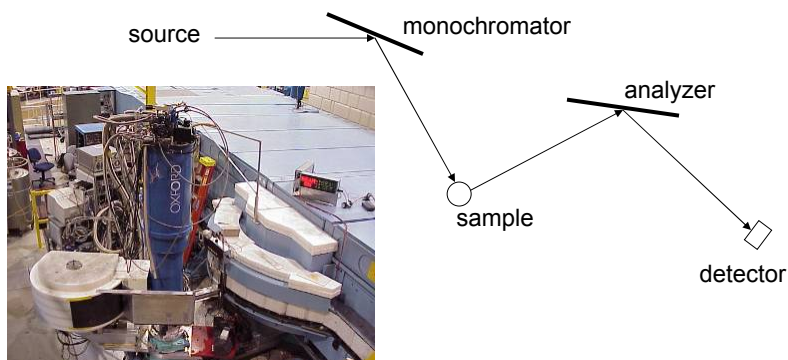
How is QENS measured?

- Triple axis spectrometer (TAS)
- Time-of-flight spectrometer (TOF)
- Backscattering spectrometer (BS)
- Neutron spin echo spectrometer (NSE)

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Triple axis spectrometer

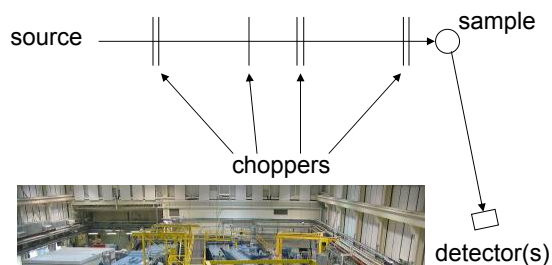


The "SPINS" spectrometer at NIST
Incident energy 14 ~ 2 meV (2.4 ~ 6.1 Å)
Flux at sample: 3.9×10^6 n/cm²/s at 4 Å.
Resolution: from 0.02 to 1.0 meV

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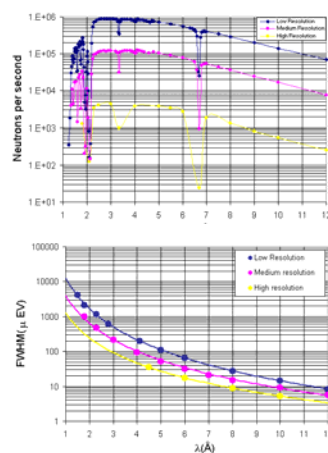
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Time-of-flight spectrometer



J.R.D. Copley and J.C. Cook,
Chem. Phys. **292**, 477 (2003)

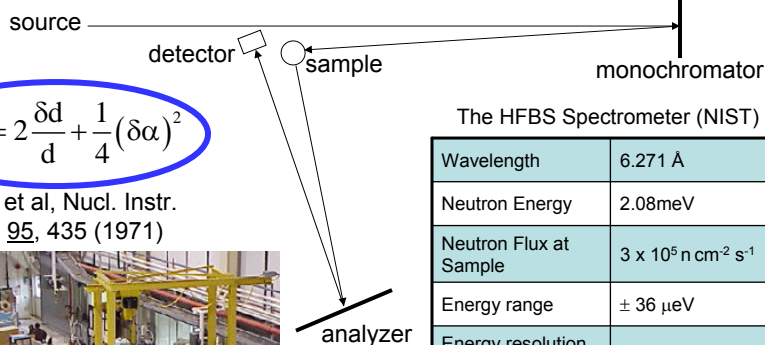
The Disk Chopper Spectrometer (NIST)



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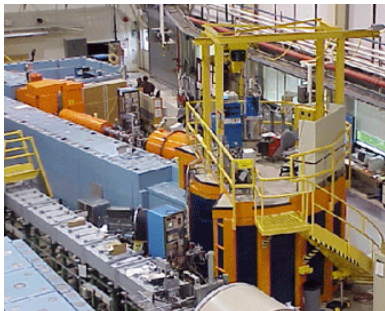
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Backscattering spectrometer



$$\frac{\delta E}{E} = 2 \frac{\delta d}{d} + \frac{1}{4} (\delta \alpha)^2$$

M. Birr et al, Nucl. Instr.
Meth. **95**, 435 (1971)



The HFBS Spectrometer (NIST)

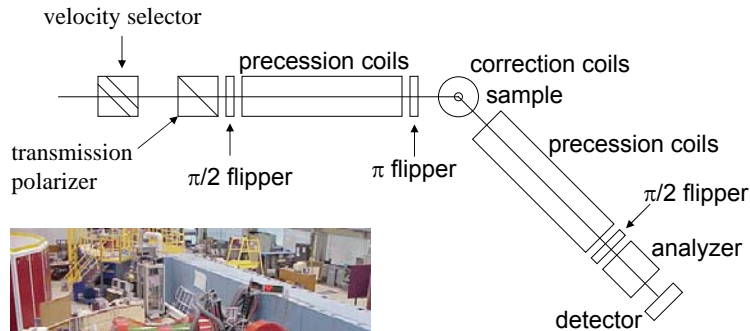
Wavelength	6.271 Å
Neutron Energy	2.08 meV
Neutron Flux at Sample	$3 \times 10^5 \text{ n cm}^{-2} \text{ s}^{-1}$
Energy range	$\pm 36 \text{ } \mu\text{eV}$
Energy resolution at $\pm 36 \text{ } \mu\text{eV}$	About $1 \text{ } \mu\text{eV}$
Analyzer Span	165°
Q range	$0.25 \text{ } \text{\AA}^{-1} - 1.75 \text{ } \text{\AA}^{-1}$

A. Meyer, et al, Rev. Sci. Instrum.,
74, 2759 (2003).

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Spin echo spectrometer



NSE measures $I(Q,t)$ for times t between ≈ 5 ps and 100-400 ns.
(<http://www.ncnr.nist.gov/instruments/nse/>)

After D.A. Neumann and B. Hammouda, J. Res. NIST **98**, 89 (1993)

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Spin echo spectrometer

IN WORDS

The spin echo technique uses the precession of a neutron's magnetic moment (spin) in a magnetic field as a "clock" to measure the neutron's speed.

The neutron spins undergo many (of order 10^5) turns in the precession coils.

In effect each spin is "wound up" in the first coil and "unwound" by the same amount in the second coil when the scattering is elastic.

If the scattering is inelastic, the "winding up" and "unwinding" processes do not completely cancel. The polarization of the neutron beam at the echo position is a measure of the inelasticity of the scattering.

IN EQUATIONS

$$\phi = \gamma \frac{J_0}{v_0} \quad (J_i = \int B_i dl) \quad (\omega_L = \gamma B)$$

$$[\gamma = 1.83247185(43) \times 10^8 \text{ rad/s/T}]$$

$$\Delta\phi = \gamma \frac{J_0}{v_0} - \gamma \frac{J_1}{v_1} \approx \gamma \frac{J_0}{v_0} \left(\frac{\delta v}{v_0} \right) - \gamma \frac{\delta J}{v_0}$$

At the echo condition δJ vanishes.

$$\hbar\omega = \frac{1}{2} m \delta(v_0^2) = m v_0 \delta v ; v_0 = \frac{h}{m\lambda}$$

$$\Delta\phi \approx \gamma \frac{J_0}{v_0} \frac{\hbar\omega}{m v_0} = t_F \omega ; t_F = \gamma \frac{J_0 \hbar}{m v_0^3}$$

$$P_x = \langle \cos(\Delta\phi) \rangle ; t_F = \gamma \frac{m^2}{2\pi\hbar^2} J_0 \lambda^3$$

$$= \frac{\int S(Q, \omega) \cos(t_F \omega) d\omega}{\int S(Q, \omega) d\omega} = \frac{I(Q, t_F)}{I(Q, 0)}$$

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Comparison of spectrometers

• Resolution

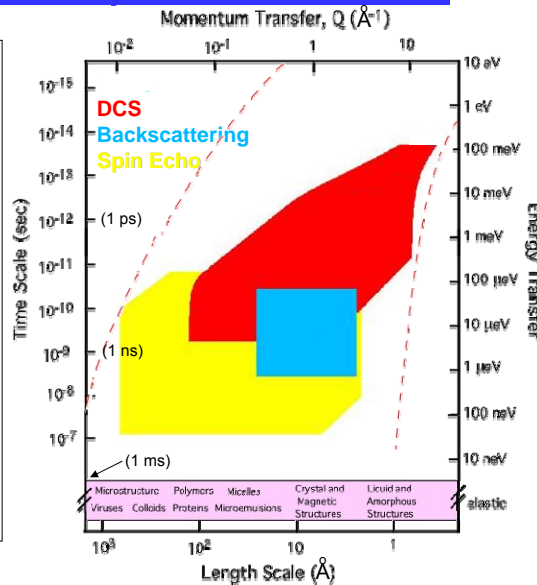
TAS → TOF → BS → NSE
Good → Very good → Excellent

• Q and ω coverage

TOF has wide coverage in Q and ω ,
BS has wide Q coverage (but poor Q
resolution), TAS/NSE are more
restricted. BS has restricted ω range.

• Coherent and incoherent scattering

NSE is best suited to measurements of
coherent diffusive or dispersionless
excitations at long times. Other forms
of dynamical behavior (in the same
time range) can be studied, but they are
less straightforward.



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How is QENS modeled?

Separation of motions with different time scales

Vibrations (internal, external), translations

Local motions (local diffusion, reorientations)

Translational diffusion

It is generally assumed that motional time scales differ so much that the
coupling between different types of motion can be neglected.

For example, consider vibrations and diffusion :

$$I(Q, t) = I_v(Q, t) \cdot I_D(Q, t)$$

$$S(Q, \omega) = S_v(Q, \omega) \otimes S_D(Q, \omega)$$

In this case the approximation is justified. On the other hand an equivalent
approximation, separating translational and rotational diffusive motions, is less
satisfactory.

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Detailed balance and moment rules

$S(Q, \omega)$ should ideally satisfy detailed balance and the moment sum rules.

Moments

$$\langle \omega^0 \rangle = \int_{-\infty}^{\infty} S(Q, \omega) d\omega = 1$$

Detailed balance

$$S(Q, -\omega) = e^{-\hbar\omega/k_B T} S(Q, \omega)$$

$$\langle \omega^1 \rangle = \int_{-\infty}^{\infty} \omega S(Q, \omega) d\omega = \frac{\hbar Q^2}{2M}$$

Most model $S(Q, \omega)$'s do not satisfy these rules. $\langle \omega^2 \rangle = \int_{-\infty}^{\infty} \omega^2 S(Q, \omega) d\omega \approx \frac{Q^2 k_B T}{M}$

At sufficiently large Q and ω the ideal gas scattering function applies.

$$\begin{aligned} S_{IG}(Q, \omega) &= \frac{1}{\sqrt{2\pi} \sigma} \exp\left[-\frac{(\omega - \Delta)^2}{2\sigma^2}\right] \text{ with } \Delta = \frac{\hbar Q^2}{2M}, \sigma = \sqrt{\frac{Q^2 k_B T}{M}} \\ &= \frac{1}{\sqrt{2\pi} \sigma} \exp\left[-\frac{\Delta^2}{2\sigma^2}\right] \exp\left[-\frac{\omega^2}{2\sigma^2}\right] \exp\left[\frac{\hbar\omega}{2k_B T}\right] \end{aligned}$$

$S_{IG}(Q, \omega)$ is Gaussian: unit area, width $\propto Q$, shift $\propto Q^2$

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Popular models

Long range continuous diffusion

$$I(Q, t) = \exp(-DQ^2 t)$$

and $S(Q, \omega)$ is a Lorentzian:

$$S(Q, \omega) = \frac{1}{\pi} \frac{\Delta\omega(Q)}{\omega^2 + [\Delta\omega(Q)]^2}$$

$$\text{with } \Delta\omega(Q) = DQ^2$$

Long range jump diffusion

(G.T. Chudley and R.J. Elliott, Proc. Phys. Soc. **77**, 353 (1961))

$$S(Q, \omega) \text{ is a Lorentzian with } \Delta\omega(Q) = \frac{1}{Z\tau} \sum_{j=1}^Z \left(1 - \exp[-\mathbf{Q} \cdot \mathbf{s}_j]\right)$$

(lattice)

$$\Delta\omega(Q) = \frac{6D_s}{r^2} (1 - j_0[Qr])$$

(isotropic)

Spatially confined continuous diffusion on a sphere, radius r

(V.F. Sears, Can. J. Phys. **44**, 1999 (1966))

$$\begin{aligned} S(Q, \omega) &= j_0^2(Qr) \delta(\omega) + \sum_{\ell=1}^{\infty} j_{\ell}^2(Qr) \times \\ &\times (2\ell+1) \frac{1}{\pi} \frac{\ell(\ell+1)D_R}{[\ell(\ell+1)D_R]^2 + (\hbar\omega)^2} \end{aligned}$$

Spatially confined jump diffusion

(2 sites, separation d)

$$\begin{aligned} I(Q, t) &= A_0(Q) + [1 - A_0(Q)] \exp(-2t/\tau) \\ S(Q, \omega) &= A_0(Q) \delta(\omega) + [1 - A_0(Q)] \frac{1}{\pi} \frac{\Gamma}{\omega^2 + \Gamma^2} \\ A_0(Q) &= \frac{1}{2} [1 + j_0(Qd)], \Gamma \text{ independent of } Q \end{aligned}$$

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The Elastic Incoherent Structure Factor (EISF)

$$S(Q, \omega) = A_0(Q) \delta(\omega) + S_{\text{inel}}(Q, \omega)$$

$$\int_{-\infty}^{\infty} S(Q, \omega) d\omega = 1 \quad A_0(Q) = I(Q, \infty) = \frac{1}{2\pi} \int_{-\infty}^{\infty} G(r, \infty) \exp(-iQ \cdot r) dr$$

$$A_0 = \frac{\text{elastic}}{\text{elastic} + \text{inelastic}} \quad A_0(Q) = \frac{1}{N^2} \left| \sum_j \exp(iQ \cdot r_j) \right|^2 \quad \text{See, e.g., Yildirim et al, PRB 60, 314, (1999).}$$

- For long-range diffusion $A_0=0$ (except at $Q=0$)
- For both long-range and restricted diffusion $A_0 = 1$ at $Q=0$
- For a continuous set of sites $A_0 \rightarrow 0$ at large Q
- For a discrete set of sites $A_0 \rightarrow >0$ at large Q , e.g. $A_0 \rightarrow 1/N$ for N equivalent sites.

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Spatially confined jump diffusion

(1) Jumps among 2 sites (site separation d)

$$S(Q, \omega) = \frac{1}{2} [1 + j_0(Qd)] \delta(\omega) + \frac{1}{2} [1 - j_0(Qd)] \frac{1}{\pi} \left(\frac{2\tau}{4 + \omega^2 \tau^2} \right)$$

τ is average time between jumps

(2) Jumps among 3 sites on a circle of radius r

$$S(Q, \omega) = \frac{1}{3} [1 + 2j_0(Qr/\sqrt{3})] \delta(\omega) + \frac{2}{3} [1 - j_0(Qr/\sqrt{3})] \frac{1}{\pi} \left(\frac{3\tau}{9 + \omega^2 \tau^2} \right)$$

(3) Jumps among N sites on a circle of radius r

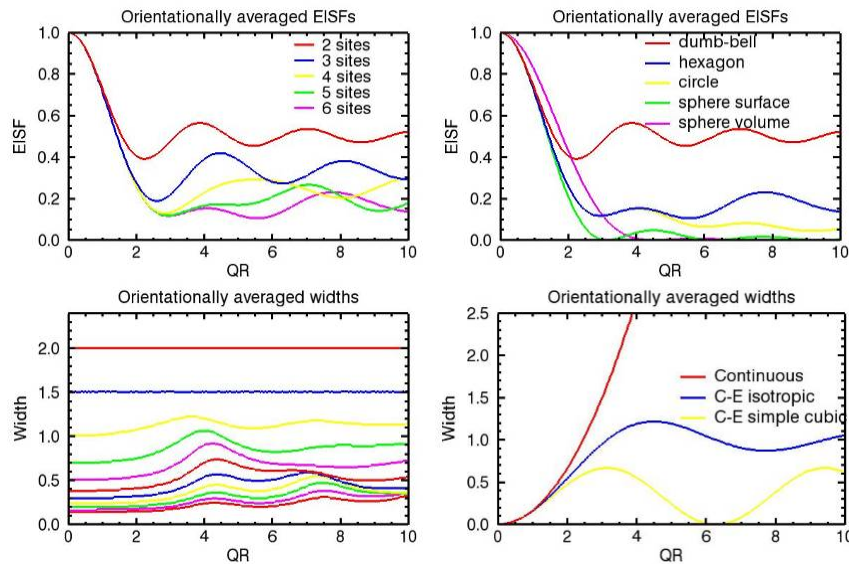
$$S(Q, \omega) = A_0(Q) \delta(\omega) + \sum_{\ell=1}^N A_{\ell}(Q) \frac{1}{\pi} \left(\frac{\tau_{\ell}}{1 + \omega^2 \tau_{\ell}^2} \right) \quad \tau_{\ell}^{-1} = 2\tau^{-1} \sin^2 \left(\frac{\pi \ell}{N} \right)$$

$$A_{\ell}(Q) = \frac{1}{N} \sum_{n=1}^N j_0(Qr_n) \cos \left(\frac{2\ell n \pi}{N} \right) \quad r_n = 2r \sin \left(\frac{n\pi}{N} \right)$$

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Examples of EISFs and widths



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Practicalities ("caveat utor")

• Planning an experiment

- Choice of spectrometer: match instrument/instrument setup (e.g. choice of wavelength) to time scale(s) of interest
- Choice of sample shape: typically annular for TOF and BS - flat plate for TAS and NSE
 - Both geometries can be challenging, esp. annular powders, since optimum thicknesses are of order tenths of a mm
- Choice of sample dimensions (consider self-shielding, multiple scattering)

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Choice of spectrometer

Matching the instrument to time scale(s) of interest

← (slow) ——— $S(Q, \omega)$ ——— (fast) →

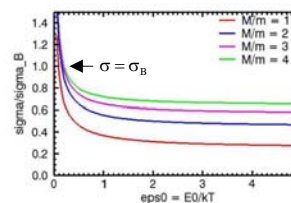
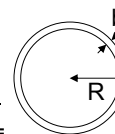
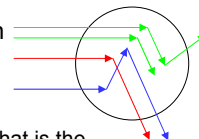
		delta-function peak	Narrow peak	Medium width peak	Broad peak	Flat back-ground
Resolution ↑ ↓	Low resn. (broad)	(Elastic)	Elastic	Elastic	Match	(Flat)
	Med. resn. (medium)	(Elastic)	Elastic	Match	Flat	(Flat)
	High resn. (narrow)	(Elastic)	Match	Flat	Flat	(Flat)

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Multiple scattering and self-shielding

- For slab geometry self-shielding (SS) depends strongly on orientation; for annular geometry it is almost isotropic
- The 90% transmission “rule of thumb”
 - The “rule” is that if $T=90\%$, $S_1=10\%$ and $S_m/S_1=10\%$, and if that is the case multiple scattering (MS) can be neglected.
 - The first part is valid for slab geometry, not necessarily valid for other geometries: for example, consider annular geometry with $b=0.1$ mm and $R=10$ mm, i.e. $b/R = 0.01$; if $T=90\%$, $S_1=8.5\%$ and $S_m/S_1 \approx 18\%$.
 - If indeed $S_m/S_1 = 10\%$, can neglect of MS be justified? What if $S_m/S_1 = 15\%$?
- If T is calculated what value should be used for σ_s ?
 - In general σ_s depends on E_i , T , chemistry, morphology, ... See J.R.D. Copley, Neutron News, 18(1), 30 (2007).
- An excellent reference for MS/SS is V.F. Sears, Adv. Phys. 24, 1 (1975).



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Multiple scattering

Quotes from early papers

"The average of distributions obtained at small angles of scattering ($Q < 1.4$) was taken to be the **multiple scattering** component, ..."

"The thickness of the water films ranged from 0.01 inches (~0.08 M.F.P.) to 0.035 inches (~0.3 M.F.P.) depending on the amount of **multiple scattering** which could be tolerated. ... **Multiple scattering** was estimated roughly [ref.] as shown ..."

Quotes from more recent papers

"Sample thicknesses were kept around 0.2 mm to achieve transmissions of $\approx 90\%$ and avoid **multiple scattering** effects."

"The sample thickness was chosen to ensure **90%** neutron transmission and thus minimize **multiple scattering** effects."

"... sample holders chosen to ensure greater than **90%** neutron beam transmission through the sample in order to minimize the effects of **multiple scattering** ..."

"The **multiple scattering** was minimized using a small thickness for the sample, which was also confirmed by a transmission higher than **0.9**."

"Total neutron scattering from the samples was $\sim 10\%$; thus, **multiple scattering** was negligible, ..."

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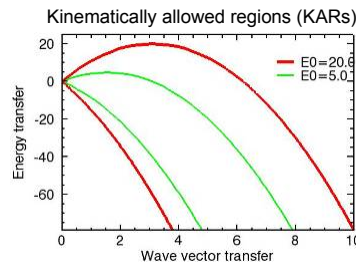
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Total scattering cross sections

- Total scattering cross sections vary with incident energy, temperature, morphology, and chemical environment (not just chemical composition)
- Total scattering cross sections vary as $1/v$ at sufficiently long wavelengths (as do absorption cross sections)

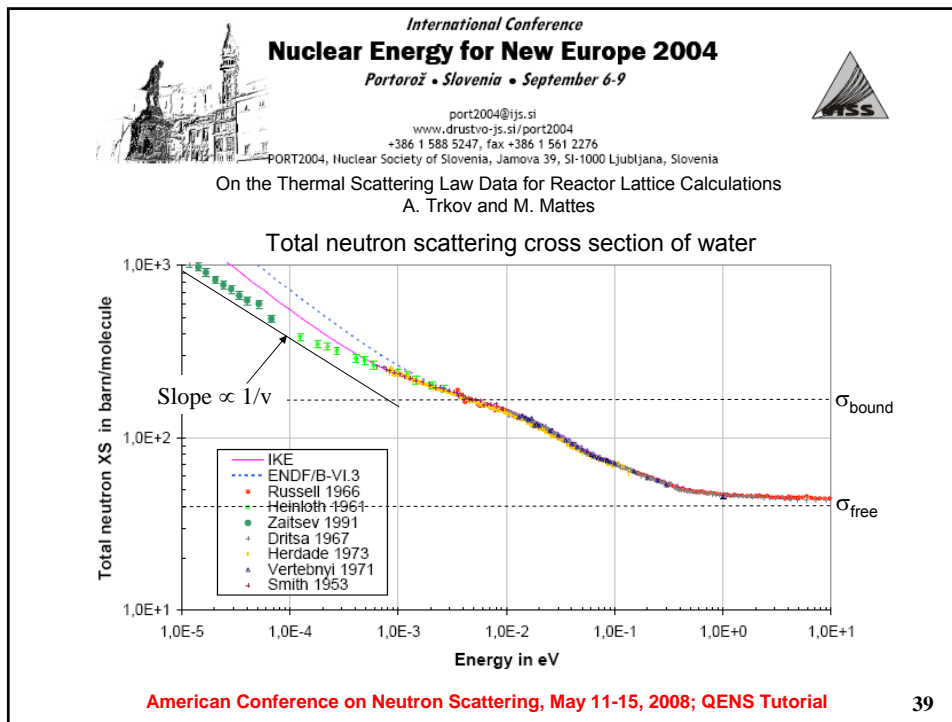
$$\sigma_s(E_0) = \int_{4\pi} \int_0^\infty \left(\frac{d^2\sigma}{d\Omega dE_f} \right) dE_f d\Omega$$

$$\propto \frac{1}{E_0} \int_{\text{KAR}(E_0)} S(Q, \omega) Q dQ d\omega$$



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Practicalities (contd.)

- **Performing the experiment**
 - What to run, apart from the sample(s), and how to divide the time?
 - Empty can? Empty cryostat/CCR?
 - Elastic scatterer (for detector efficiencies, normalization, resolution)?
 - Time-independent background (“dark count”)?
 - Choices of wavelength, resolution, dynamic range etc.
- **Data treatment (“postparation”)**
 - The options available for data treatment depend on how the experiment was planned and performed.
 - Keep in mind:
 - the resolution-limited “elastic” scattering may include elastic scattering from host material in the sample, and other sources, as well as QENS due to motions too slow to be distinguished from truly elastic scattering
 - The flat “background” may include true background as well as QENS from motions too fast to be detected as such
 - When subtracting the empty can, consider the attenuating effect of the sample

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Examples of QENS

- Water
- Cubane
- Liquid lead
- C₆₀

Liquid water

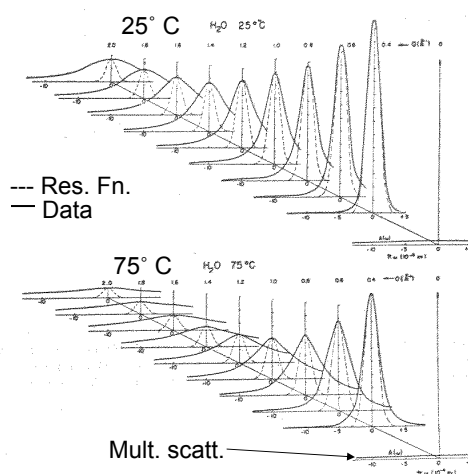


Fig. 1. The scattering function $S(Q, \omega)$ for water (a) at 25°C and (b) at 75°C for small values of Q . The energy resolution functions are shown as dashed curves. The multiple scattering $A(\omega)$ which was subtracted is also shown.

M. Sakamoto et al, Proc. Int. Conf. Magn. and Cryst. 1961, vol. II; J. Phys. Soc. Japan 17, Suppl. B-II, 370 (1962).

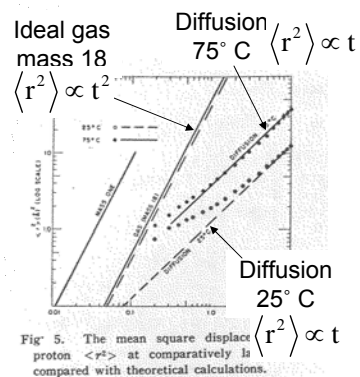
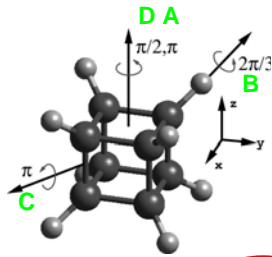
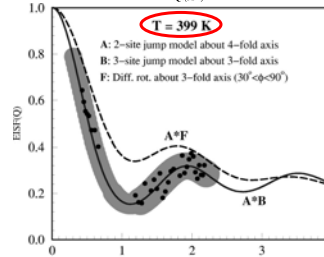
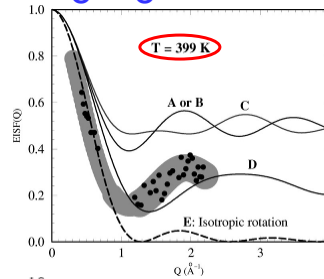
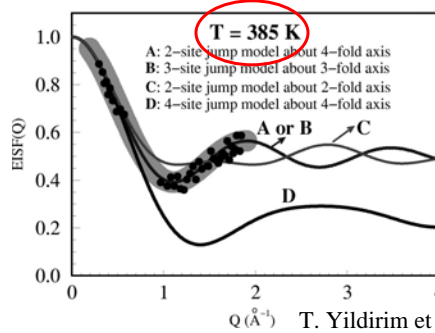


Fig. 5. The mean square displacement $\langle r^2 \rangle$ at comparatively is compared with theoretical calculations.

Cubane, C_8H_8



Phase transition,
from orthorhombic
($\alpha=73^\circ$) to
orthorhombic
($\alpha=103^\circ$) at 394K.
Melting at 405K



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$S(Q, \omega)$

Liquid lead

B.N.Brockhouse and N.K.Pope, PRL 3, 259(1959)

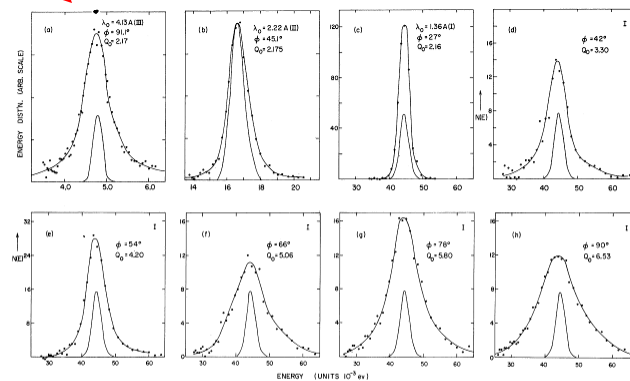
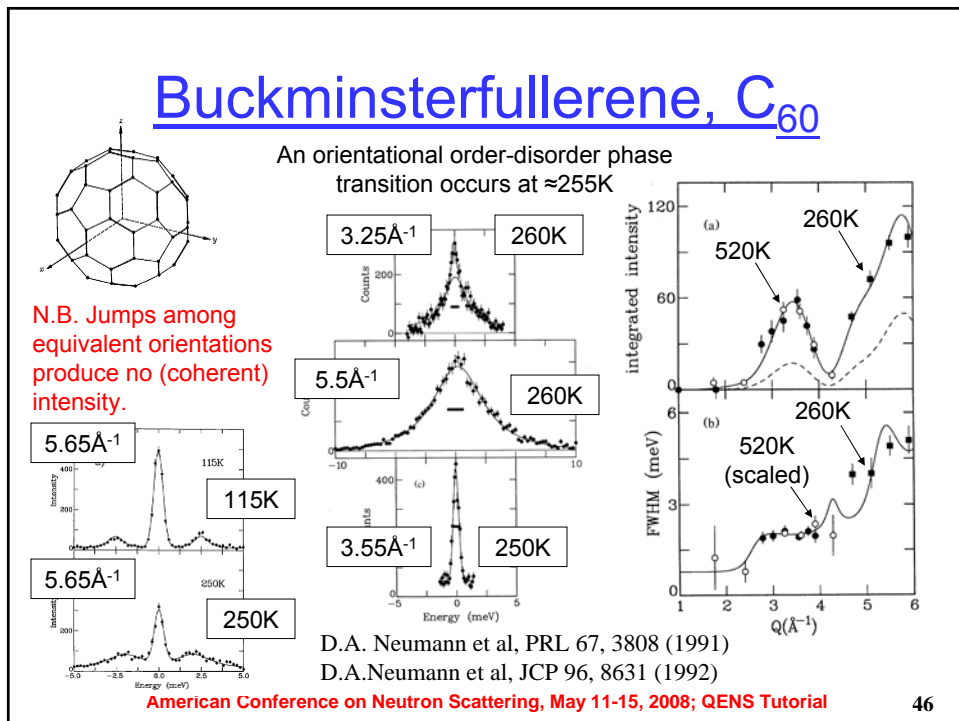
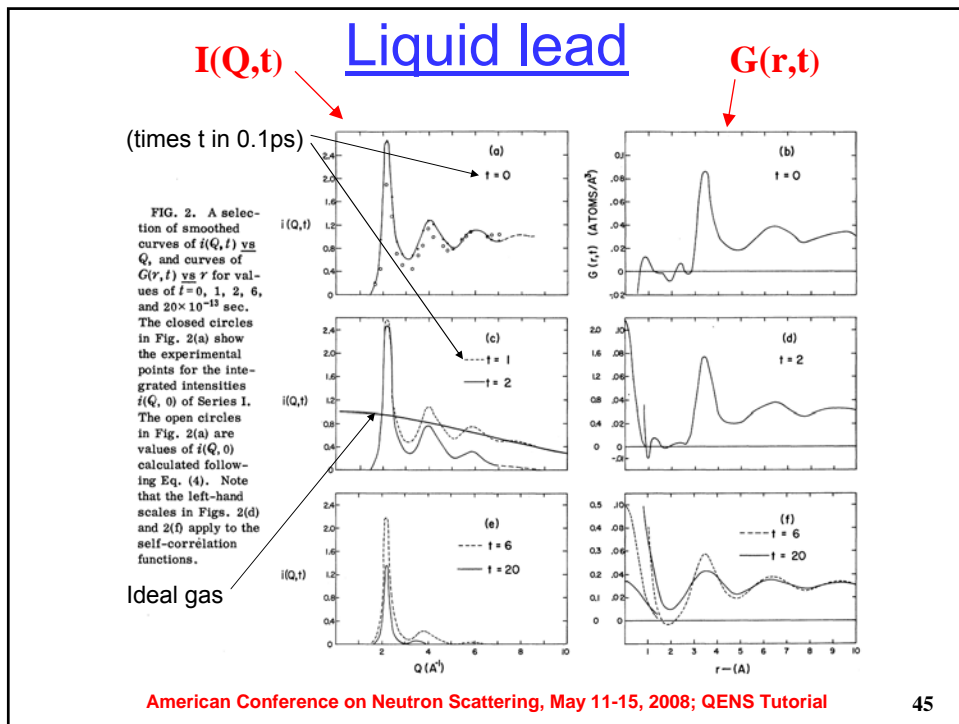


FIG. 1. A selection of energy distributions of neutrons scattered from liquid lead at 620°K, each with its resolution function (appropriate to the incident energy). All patterns have been corrected for background, for container scattering, for instrument sensitivity, and all except (a) for multiple scattering. [Figure 1 (a) is thought to contain ~ 10 % multiple scattering.] The factor k'/k_0 of Eq. (1) has also been removed. Figures 1 (a), (b), and (c) show energy distributions obtained under different resolutions at the main diffraction peak. In Fig. 1 (b) the resolution function has been normalized at the peak to illustrate the energy broadening. Figures 1 (a) and (b) are in agreement that the half-width at half maximum of the distribution is $\sim 3.5 \times 10^{-4}$ eV, compared with the 8×10^{-4} eV expected on simple arguments of diffusion broadening by small motions [references 2 and 3]. The shape is roughly Lorentzian. Figures (c) to (h) show Series I distributions at different values of Q_0 . The intensities are consistently normalized but note the different ordinate scales.

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Further reading

- M. Bée, “Quasielastic Neutron Scattering ... in Solid State Chemistry, Biology and Materials Science” (Adam-Hilger, Bristol 1988) (but difficult to find)
- R. Hempelmann, “Quasielastic Neutron Scattering and Solid State Diffusion” (OUP, Oxford and New York, 2000)
- J.S. Higgins and P. Benoit, “Polymers and Neutron Scattering” (OUP, Oxford and New York, 2000)
- J. Fitter, T. Gutberlet, and J. Katsaras, eds., “Neutron Scattering in Biology” (Springer, Berlin, 2006)
- Proceedings of QENS conferences (next slide)

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Recent QENS Conferences/Proceedings

- **QENS 1992, 1994, 1996, 1998,...**
- **QENS 2000:** Physica B: 301, Issues 1-2, July 2001
- **QENS 2002:** Chemical Physics 292, Issues 2-3, (2003)
- **QENS 2004:** Phys. Chem. Chem. Phys. 7 (2005); go to <http://blake.ism.u-bordeaux1.fr/A.Desmedt/qens2004.html>
- **QENS 2006:** “Quasi-Elastic Neutron Scattering Conference 2006 (QENS2006)”, ed. P.E. Sokol, H. Kaiser, D. Baxter, R. Pynn, D. Bossev, M. Leuschner (Mat. Res. Soc., Warrendale, PA, 2007)
- **QENS 2009:** to be held 2/10-13/09 at the Paul Scherrer Institut, Switzerland.

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